RENORMALIZATION EXAMPLE IN $D=4$

free fields

Calculate 1-loop effective action, div. pieces local:

$$I_{eff} = \int d^4x \sqrt{g}\left[\frac{R}{16\pi G} + \alpha R^2 + \beta (R_{ab})^2 + \gamma (R_{abcd})^2\right]$$

$$+ I_{nonlocal}$$

$$\Delta \frac{1}{G} = f_G\, \epsilon^{-2}$$

$$\Delta\alpha = f_\alpha \ln(\epsilon) + g_\alpha$$

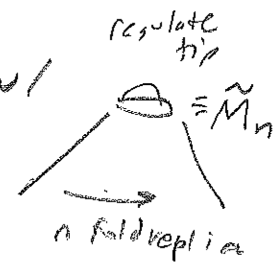
f_G, g's nonuniversal

$$\Delta\beta = f_\beta \ln(\epsilon) + g_\beta$$

$$\Delta\gamma = f_\gamma \ln(\epsilon) + g_\gamma$$

$f_{\alpha,\beta,\gamma}$ Universal (depend on # & type of species)

FPS calculated "squashed cone" entropy associated w/ regulated replicated manifold, in $\lim n \to 1$



$$\int_{\tilde{M}_n} \sqrt{g}\, d^Dx\, R \to n \int_M \sqrt{g}\, d^Dx\, R + \underbrace{8\pi(1-n)A(E)}_{\text{from tip}} + \ldots$$

$\ldots \; \mathcal{O}((1-n)^2)$

$$\int_{\tilde{M}_n} \sqrt{g}\, d^Dx\, R^2 \to n \int_M \sqrt{g}\, d^Dx\, R^2 + 8\pi(1-n) \int_E d^{D-2}y \sqrt{g}\, R + \ldots$$

$$\int_{\tilde{M}_n} \sqrt{g}\, d^Dx (R_{ab})^2 \to n \int_M \boxed{} + 4\pi(1-n) \int_E d^{D-2}y \sqrt{g}\left(R_{ii} - \tfrac{1}{2} K_{ii} K_{jj}\right)$$

$$\int_{\tilde{M}_n} \sqrt{g}\, d^Dx (R_{abcd})^2 \to n \int_M \boxed{} + 8\pi(1-n) \int_E d^{D-2}y \sqrt{g}\left(R_{ijij} - K_{ij}K_{ij}\right) + \ldots$$

D=4 CONT'D

$$S_{gen} = \langle S_{grav} \rangle + S_{out}$$

$$= \lim_{\epsilon \to 0} \Big[S_{out}^{(\epsilon)} + \frac{A}{4G(\epsilon)\hbar} + \frac{8\pi}{\hbar} \int_E d^2x \sqrt{{}^2g} \Big[$$

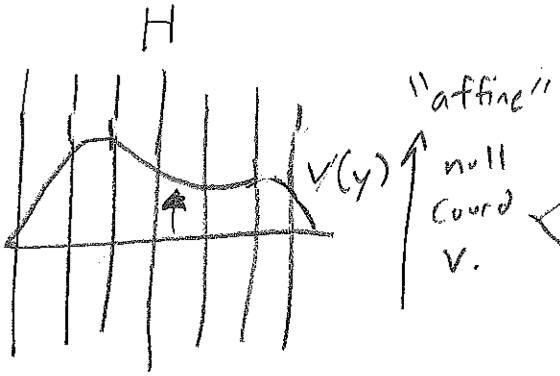
$$\alpha(\epsilon) R + \frac{\beta(\epsilon)}{2}\left(R_i{}^i - \frac{1}{2} K_i{}^{ia} K^j{}_{ja}\right) + \gamma(\epsilon)\left(R_{ij}{}^{ij} - K_{ij}{}^a K_a{}^{ij}\right) \Big]$$

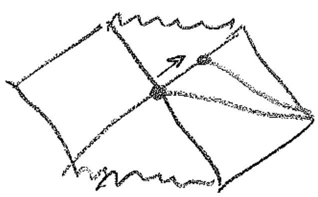
where the EE formula from earlier came from

DISCUSS CONTACT TERMS?

MONOTONICITY PROPERTIES OF S_{gen}

GSL | Increases on Causal Horizon





2nd Law requires comparing two time slices of horizon.

Can evolve forwards in wiggly way.

assume 1st order pert. from T_{ab}, semiclassical

Calculate area by focussing of lightrays from future

$$A(\infty)-A(v') = \int_{v'} (v-v')\, T_{vv}\, dv$$

for $v'=0$ (bif surface) flux of Killing energy across horizon when v' = bif surface but still valid for other choices of v'.

(obtained from linearized Raychaudhuri Eq.

$$\dot{\theta} = -\frac{\theta^2}{D-2} - \sigma_{ab}\sigma^{ab} - R_{ab}k^a k^b \quad)$$

$$\theta = \frac{1}{A}\frac{d\theta}{d\lambda}$$

σ_{ab} = shear

Use monotonicity of rel S:

$$S(\rho|\sigma)_{V_2} \le S(\rho|\sigma)_{V_1}$$

State you care about — HH thermal State

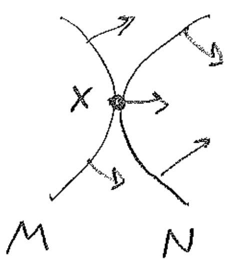
need to show state above cut v' is always thermal $e^{-2\pi K}$ where K generates dilations about slice.

Possible to show thermality by lightfront quantization (fine print)

BEND MONOTONICITY

S.2

N not inside of M but touching at X

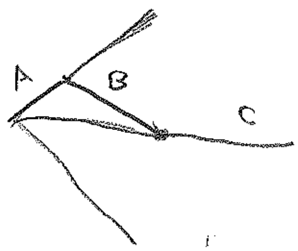


$$\left.\frac{\delta S_{gen}}{\delta v(x)}\right|_M \geq \left.\frac{\delta S_{gen}}{\delta v(x)}\right|_N$$

(at least semiclassically)

1) θ piece obvious by geometry

2) $\frac{\delta}{\delta v} S_{out}$ piece from monotonicity of mutual info = SSA since adding extra region only makes S_{gen} increase faster.



$$S_{BC} - S_{ABC} \geq S_B - S_{AB}$$

3) Use $\hbar G$ expansion to neglect higher curv. entropy in comparison w/ area piece.

QUANTUM FOCUSSING

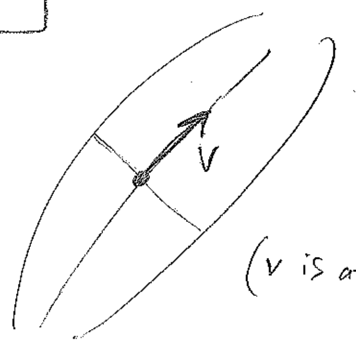
INFO LOSS

D:

ENTROPY FOCUSES

Bousso, Fisher, Leichenauer, Wall "A quantum focussing conjecture"
+ Koeller "Proof of the Quantum Null Energy Condition"
Wall "A Second Law for higher curvature gravity"
Bhattacharjee, Sarkar, Wall, "The holographic entropy increases in higher curvature gravity"
Wall "The Generalized Second Law implies a Quantum Singularity Theorem"

$\boxed{\text{FOCUSING OF AREA}}$



(v is affine)

Raychaudhuri Equation says area focusses

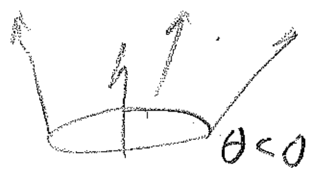
$$\frac{d\theta}{dv} = -\frac{\theta^2}{D-2} - \sigma_{ab}^2 - R_{vv} \qquad \theta = \frac{1}{A}\frac{dA}{dv}$$

$\Downarrow$ + Einstein $R_{vv} = 8\pi T_{vv}$

+ NEC $T_{vv} \geq 0$

$$\dot{\theta} \leq 0$$

Penrose singularity thm uses this + G.H. + space noncompact to prove that singularities must form if trapped surface: compact surface whose lightrays initially contract going outwards,



$\theta < 0$

$\theta \to -\infty$ @ v = finite

Also places many other important constraints on spacetimes:

- no wormholes or warp drives, AdS has boundary causality
- restrictions on baby universes, can't restart inflation
- initial singularity for noncompact FRW-like universes

These are important results → like to generalize or circumvent
→ but do they hold behind Einstein gravity?

② BH entropy

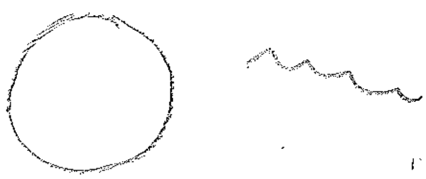
event horizons of black holes (also Rindler, de Sitter & other causal horizons) seem to obey laws of thermo if you assign

$$S_{BH} = \frac{A}{4G\hbar}$$ increases classically if NEC (Hawking)

(believed to count QG microstates)

there are corrections beyond Einstein gravity, of 2 kinds

1) Quantum / thermal corrections



$$S_{out} = -\mathrm{tr}(\rho \ln \rho)$$

counts entropy of matter outside

important in quantum situations e.g. Hawking radiation where sum

$$S_{gen} = S_{BH} + S_{out}$$

is increasing: (GSL) $\rightarrow$ $\boxed{\frac{\delta}{\delta v} S_{gen} \geq 0}$

proven semiclassically Wall || for free / superren. fields minimally coupled to Einstein using lightfronts & mono. of rel S

S_{out} includes a piece coming from entanglement entropy near horizon, which is divergent & must be renormalized

2) higher curvature (counterterms / stringy) corrections

$$I = \int d^D x \sqrt{-g} L, \qquad L = \frac{R}{16\pi G} + \alpha (R^2 \ldots$$

$\Downarrow$ corrections to BH entropy

$$S_{BH} = -\frac{2\pi}{\hbar} \int d^{D-2}x \sqrt{g} \; \underbrace{4 \frac{\partial L}{\partial R_{uvuv}}}_{\text{Wald, valid stationary, } f(R)} + \underbrace{16 \frac{\partial^2 L}{\partial R_{uiuj} R_{vkvl}} K^{(u)}_{ij} K^{(v)}_{kl} + O(K^4) + \ldots}_{\text{Solodukhin, FPS, Dong, Camps, Miao, ..}}$$

$f(\text{Riemann})$

$\cancel{\chi^u_v}$ $g_{uv} = 1$, ij trans.

divergences from S_{out} renormalize $\frac{1}{G}$, α in a consistent way

③ Spacetime thermodynamics

MAYBE NOT SAY:

A/4Għ term may still refer to QG microstates (Bianchi-Myers 12) or something more subtle like diff. ent.

Evaluate S_{gen} for non-horizon slices

(must be codim 2 split of Cauchy slice)

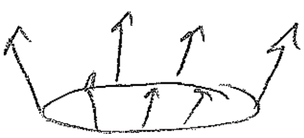
Research program: extend classical GR results to semiclassical gravity by replacing A w/ S_{gen} everywhere

(Wall 11b, FLM13, Engelhardt + Wall 14, Bousso-Engelhardt, ...)

↓

GSL + G.H. ⟹ quantum singularity thm (w/o NEC) ⟹ refer back to list

(e.g. at least semiclassically, spatially noncompact bounces impossible)



"quantum trapped surface"

$$\Theta \equiv \frac{\delta S_{gen}}{\delta V} = \frac{1}{A}\frac{\partial S_{gen}}{\partial V}\Big|_A \leq 0 \text{ everywhere}$$

proof rather global...

is there analogue of focussing thm?

$$\text{QFC}: \frac{\delta \Theta}{\delta V} \leq 0$$ (related to Bousso bound).

α must be small "string or planck scale for thy. to be causal (Camanho et al. 14)

will give partial proofs, in semiclassical regime (ħ, α expansion).

corrections important only if null surface is quasi-stationary, or else classical Einstein focussing dominates

(for "adiabatic processes", $\delta S_{gen} \approx 0$ and all terms matter)

(4)

Higher curvature corrections

comment?

start w/ covariant action with finite α corrections

$$I = I_{grav}[g^{ab}, R_{abcd}; \nabla R \ldots \phi, \nabla\phi] + I_{mat}[g^{ab}, \psi]$$

arbitrary h.c. scalar-tensor action for grav. fields

assume minimal, NEC

EOM $\quad H_{ab} = T_{ab}, \quad H_{kk} = T_{kk} \geq 0$

assume g_{ab}, ϕ are first order perturbations to a regular bifurcate Killing horizon [a bit stronger than $\theta_{cl} = 0$]

▷ if u, v are the two null directions,

any quantity w/ $w = \#_{v's} - \#_{u's}$ e.g. $w[H_{kk}] = 2$

scales like V^{-w} on Killing spacetime,

if $w > 0$, must vanish at 0th order to be regular @ $V = 0$

using this fact, can show that:

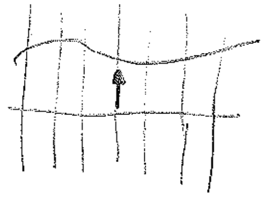
$$\delta H_{vv} = \sum_n X^{(-n)} \cdot \delta Y^{(2+n)} \Rightarrow \int_H d^{D-2}x \sqrt{g}\, T_{vv} = \frac{\hbar}{2\pi} \partial_v \partial_v S$$

- gauge fix metric so $g_{vv}, g_{vi}, g_{vv,u} = 0$ so that $w = 2$ implies at least 2 ∂_v's.
- repeatedly diff. by parts until 2 ∂_v's are outside expression, using $\partial_v[w \geq 0]\, \partial_v[w \geq 0] = 0$ @ 1st order → only δ term has $w \geq 0$

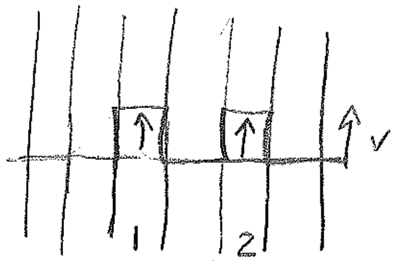
can show $S = \delta S$, for $f(\text{Riemann})$, $S = S_{Dong} + \text{total deriv.} + O(R^4)$

implies S_{Dong} obeys 2nd Law.

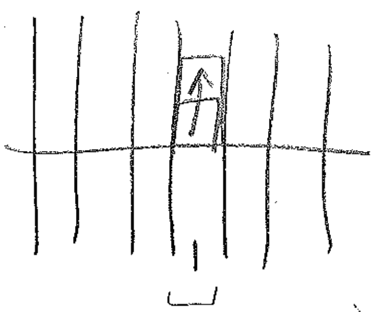
(5) <u>Quantum focussing</u>



$\frac{\delta}{\delta v}\frac{1}{A}\frac{\delta}{\delta v} S_{gen} \leq 0$



Can also consider effects of quantum fields falling across a stationary null surface – discretize into pencils.



A

divide into mixed case

$$\frac{\partial}{\partial v_1}\frac{\partial}{\partial v_2} S_{out} \leq 0 \quad \text{by strong subadditivity}$$

area does not contribute

or into "contact term"

$$\frac{\delta^2 S_{out}}{\delta v(y)\delta v(y')} = f(y)\delta(y-y')$$

+ off-diagonal

QNEC

$$\frac{\partial}{\partial v_1}\frac{\partial}{\partial v_1} S_{gen} \leq 0 \Rightarrow \langle T_{vv}\rangle \geq \frac{\hbar}{2\pi}\frac{1}{A} S''_{out}$$

stronger than SSA

restrict to 2d field theory near the vacuum (UV limit)

$$\rho = \rho_0 + \delta\rho$$

2nd order in $\delta\rho$; can be entangled w/ auxilliary system

Calculated entropy using replica trick and found it was true

n copies

- continue to nonintegen n
- $S = (1 - n\partial_n)\ln Z$
- evaluate @ $n=1$

New law of quantum information theory?

Sgen closely related to "relative entropy" (Casini)

But doesn't follow from standard properties like positivity or monotonicity since it involves a 2nd time derivative