**Holographic Entanglement Entropy**

**AdS/CFT**

Strings/M-theory on $\text{AdS}_{d+1} \times F$ \(\xrightarrow{L_{\text{AdS}} \gg L_{\text{Planck}} \text{ (i.e., } g \to 0)}\) $\xrightarrow{\text{Large } \mathcal{N}}$ CFT

Supergravity $\xrightarrow{\text{holographic CFTd}}$

**RT formula**

- Formula for leading order piece of entanglement entropy
- Applies to static or $t \to -t$ slice $\Sigma$
- People usually ignore $F$ but you don't have to!
- Find minimal area surface anchored to $E$ that divides $R$ from $\overline{R}$

Simplest example: take $\text{CFT} \times \text{CFT}$ workable

<table>
<thead>
<tr>
<th>$d$</th>
<th>CFT</th>
<th>Bulk</th>
<th>$S_{\text{strong}}$</th>
<th>$S_{\text{weak}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>D1-D5</td>
<td>$\text{AdS}_3 \times S_3 \times T_4$</td>
<td>C</td>
<td>C</td>
</tr>
<tr>
<td>3</td>
<td>ABJM</td>
<td>$\text{AdS}_4 \times S_7 (M)$</td>
<td>$N^{3/2}$</td>
<td>$N^2 (N)$</td>
</tr>
<tr>
<td>4</td>
<td>$\text{SYM}$</td>
<td>$\text{AdS}_5 \times S_5$ (JIB)</td>
<td>$N^2$</td>
<td>$N^2$</td>
</tr>
<tr>
<td>6</td>
<td>$(2,0)$ model</td>
<td>$\text{AdS}_7 \times S_4 (M)$</td>
<td>$N^3 (N)$ $\sim N^2 (IR)$</td>
<td>$N^2 (IR)$</td>
</tr>
</tbody>
</table>

$C, N$ protected by SUSY

EVIDENCE FROM UNIVERSAL PIECES & CONSISTENCY (e.g. SSA)

LM PATH INTEGRAL
Can exist multiple local minima

e.g. consider region $\mathcal{R}$ consisting of 2 disjoint intervals for $d=2$

\[ S_{AB} = S_A + S_B \]
\[ I_{A,B} = O(1) \]

let each be angle $\theta$ wide

\[ S_{AB} < S_A + S_B \]
\[ I_{A,B} = O(N^2) \]

phase transition sharp @ $O(N^2)$
but smoothed out @ finite $N$

PICTURE:
local Rindler tempering leads to deconfinement
long intervals
\[ I_{A,B} ^ {\text{long}} \sim O(N^2) \]
short intervals
\[ I_{A,B} ^ {\text{short}} \sim O(1) \]

Homology constraint crucial

related to deconfinement phase transition

low $T$$\rightarrow$ AdS

high $T$$\rightarrow$ confinement

makes it difficult to resolve
sub-AdS structure w/ RT

[MENTION 1ST LAW STUFF?]
Strong Subadditivity Proof

Very difficult to prove in q info

Easy holographically

Uses global minimization

Important consistency check

Monogamy of Mutual Info (Hayden-Hendrickson-Moloney)

\[ S(AB) + S(BC) + S(CA) \geq S(A) + S(B) + S(C) + S(ABC) \]

Only true holographically

Can be violated for general QM systems
COVARIANT VERSION

HRT
- Spacetime dynamical
- E is time dependent

Min surface makes no sense in spacetime

look for extremal surface

\[ S_{\text{ext}} = \frac{A \left[ \min \, \text{ext} \,(R) \right]}{4 \, G \cdot t} \]

still required to be homologous

Equivalent Maximin formulation:

1. On each Cauchy slice \( \Sigma \) that passes through \( E \)
2. Min area \( \Sigma \)
3. Nontrivial = NEC
   - Null curvature condition \( R_{\mu \nu \lambda \delta} g^{\mu \delta} g^{\nu \lambda} \geq 0 \)
   - Ads-hypersurfaces

\[ \text{Maximin}(R) = \min \, \text{ext} \,(R) \]

Easier to prove global results like SSA & HTHM

Can also prove that if BDA, ext surface lies deeper in bulk...