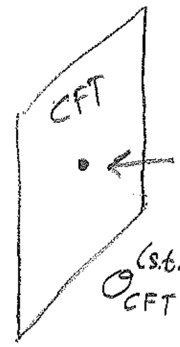
HOLOGRAPHIC ENTANGLEMENT ENTROPY

AdS/CFT

[WRITE AdS METRIC?]



$$\mathcal{O}^{(s.t.)}_{CFT} = \lim_{z\to 0} z^{-\Delta} \varphi_{bulk}$$

[CHECK THIS]

Strings/M-theory on $AdS_{d+1} \times F$ $\longleftrightarrow$ CFT_d

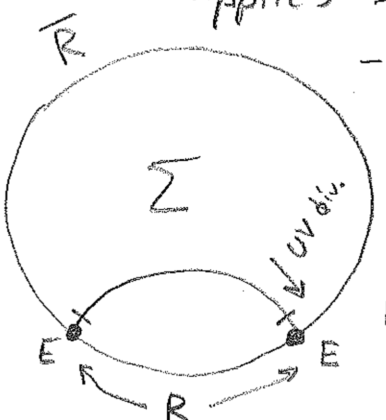
$\downarrow$ $L_{AdS} \gg \ell_{planck}$ (i.e. $G \to 0$) ($\gg \ell_s$) $\qquad\qquad$ $\downarrow$ large "N" (strongly coupled)

Supergravity $\longleftrightarrow$ holographic CFT_d

RT formula

[EXAMPLES IN VARIOUS D?]

- formula for leading order piece of entanglement entropy
- applies to static or $t \to -t$ slice Σ
  - people usually ignore F but you don't have to!



[SHOW LOCAL DIV]

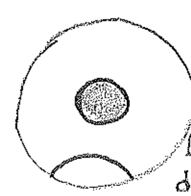
find minimal area surface anchored to E that divides R from $\bar{R}$

$$\boxed{S_{ent} = \frac{A[\min(R)]}{4G_N}}$$

$4G_N \to \ell_P^{D-2}$ (homology constraint) $\qquad$ move to next

new kind of UV regulator (bulk IR cutoff)

simplest example: take CFT × CFT wormhole



BH example

[mention deconfinement Hawking-Page phase transition]

reduces to BH entropy

examples

d	CFT	bulk	S strong	S weak
2	D1-D5	$AdS_3 \times S_3 \times T_4$	c	c
3	ABJM	$AdS_4 \times S_7$ (M)	$N^{3/2}$ $\leftarrow$	N^2 (UV)
4	$\mathcal{N}=4$ SYM	$AdS_5 \times S_5$ (IIB)	N^2	N^2
6	(2,0) model	$AdS_7 \times S_4$ (M)	N^3 (!) $\rightarrow$	N^2 (IR)

c, a protected by SUSY

$\propto 1/G + \mathcal{O}(1)$ corrections

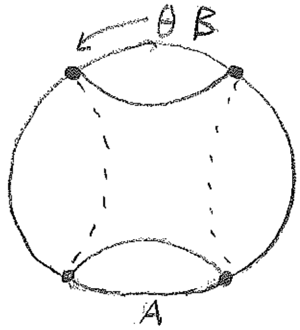
EVIDENCE FROM UNIVERSAL PIECES & CONSISTENCY (E.G SSA)

LM PATH INTEGRAL

PHASE TRANSITIONS

Can exist *multiple* local minima

e.g. consider region R consisting of 2 disjoint intervals for $d=2$



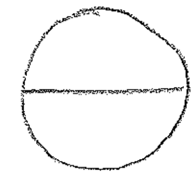
(1 conformally inv. cross ratio)

let each be angle θ wide

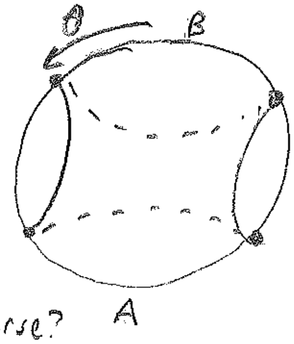
$S_{AB} = S_A + S_B$

$I_{A,B} = \mathcal{O}(1)$

[single interval conformal to sphere

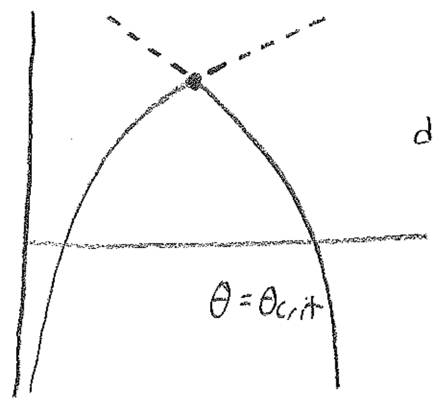


]



$S_{AB} < S_A + S_B$

$I_{A,B} = \mathcal{O}(N^2)$



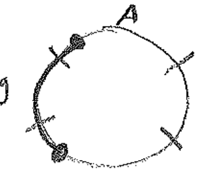
discontinuous 1st derivative

reverse?

phase transition sharp @ $\mathcal{O}(N^2)$
but smoothed out @ finite N

PICTURE:

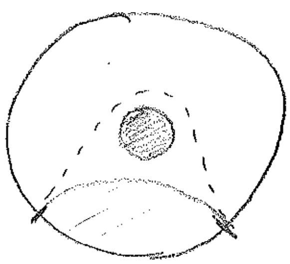
local Rindler temperature leads to deconfinement

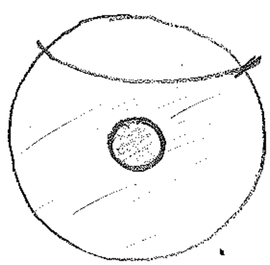


long intervals
$I_{A,B} \sim \mathcal{O}(N^2)$

short intervals
$I_{A,B} \sim \mathcal{O}(1)$

Black Hole w/ single interval





Homology constraint crucial

related to deconfinement phase transition

AdS

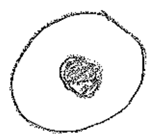
$\mathcal{N}=4$ SU(N) SUSY

low T

$\mathcal{O}(1)$

confined phase

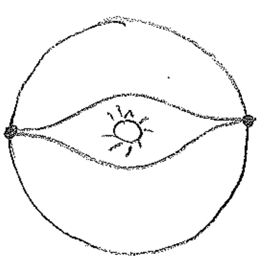
high T



$\mathcal{O}(N^2)$

quark-gluon plasma
N^2 species

Star

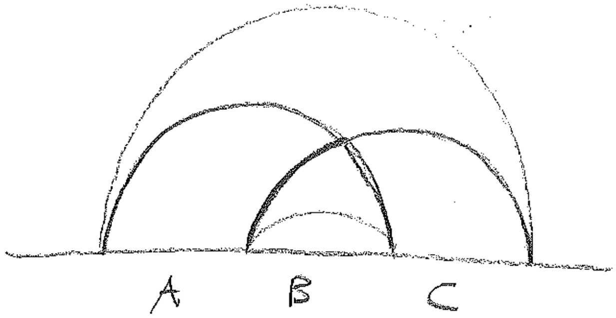


makes it difficult to resolve sub-AdS structure w/ RT

[MENTION 1ST LAW STUFF?]

[MENTION FLM?]

STRONG SUBADDITIVITY PROOF



very difficult to prove in q info

easy holographically

$$S_{AB} + S_{BC} \geq S_{ABC} + S_B$$

$$'' \geq S_A + S_C$$

uses global minimization

important consistency check

MONOGAMY OF MUTUAL INFO (Hayden-Headrick-Maloney)

$$S(AB) + S(BC) + S(CA) \geq S(A) + S(B) + S(C) + S(ABC)$$

only true holographically

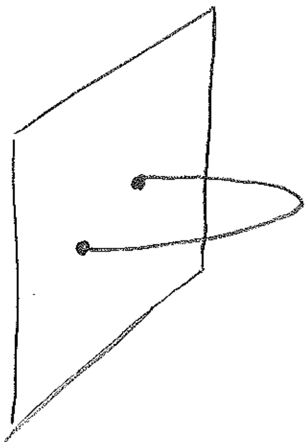
can be violated for general QM systems

[PICTURE?]

COVARIANT VERSION

HRT
- Spacetime dynamical
- E is time dependent

min surface makes no sense in spacetime

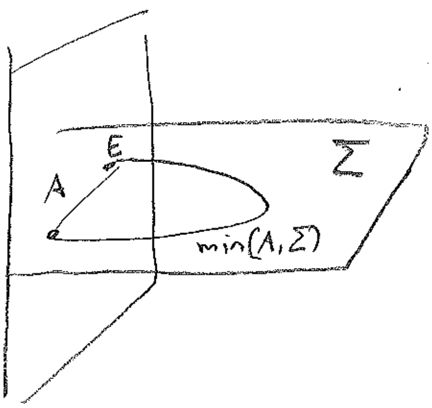


look for extremal surface

$$S_{ent} = \frac{A[\text{min ext}(R)]}{4G\hbar}$$

still required to be homologous

Equivalent Maximin formulation:

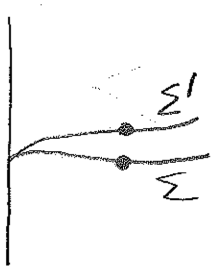


#1
on each Cauchy slice Σ that passes through E
min area surface

nontrivial

null curvature condition $R_{ab}k^a k^b \geq 0$ = NEC
+ AdS-hyperbolicity
$\Downarrow$
Maximin(R) = min ext(R)
[LOOK UP DETAILS]

choose Σ to <u>maximize</u> the value of this minimum



Easier to prove existence & global results like SSA & HHM (violated if NEC violated)

Can also prove that if $B \supset A$,
ext surface lies deeper in bulk...