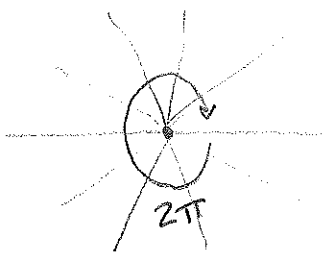
"GEOMETRIC ENTROPY"

CONICAL METHOD

valid when state is produced by a path integral with rotational $(U(1))$ invariance around E.

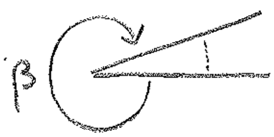


$$Z = \mathrm{tr}\left(e^{-\beta K}\right)$$

unnormalized version of $\rho = \dfrac{e^{-\beta K}}{Z}$

$\beta = 2\pi$ (usually but not required)

vary β - corresponds to introducing conical sing.



$$\frac{\partial}{\partial \beta} \overbrace{\ln Z}^{-I_{\text{eff}}} = \frac{1}{Z} \mathrm{tr}\big(\underbrace{e^{-\beta K}}_{\rho Z} \underbrace{K}_{-\beta^{-1}\ln(\rho Z)}\big)$$

$$= -\beta^{-1}\left[\mathrm{tr}(\rho \ln \rho) + \cancel{\mathrm{tr}(\rho)} \ln Z\right]$$

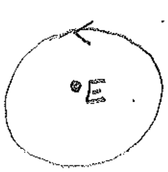
Thus

$$S(\beta') = (1 - \beta \partial_\beta) \ln Z \Big|_{\beta = \beta'}$$

linear interpolation of $\ln Z$ back to $\beta = 0$

anything linear in β drops out
(ZPE carries no entropy)

Interpretation of conical singularity

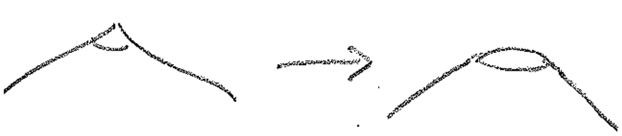


Feynman diagrams must wrap E

does not contribute

Need UV regulator (valid on curved spacetimes) to cut off small loops.

Singularity may be smeared out due to being 1st order change - not origin of UV div.



GIBBONS – HAWKING

Suppose we use Einstein Hilbert action for S_{eff}

local – no loops. Contribution linear in β except @ tip.

$$I = -\int d^4x \sqrt{-g}\, \frac{R}{16\pi G} \qquad \ln Z = -I/\hbar$$

result

$$(1 - \beta\partial_\beta) \ln Z \Big|_{\beta = 2\pi} = \frac{\text{Area}}{4G\hbar}$$

Bekenstein – Hawking entropy of horizon

Or take nonminimal scalar action

$$I = \int d^4x \sqrt{g} \left(\nabla_a\phi \nabla_b\phi + \xi \phi^2 R \right)$$

$$-\frac{\xi \langle \phi^2 \rangle}{4G\hbar}$$ contribution to EE

In a nongravity thy, this is just an ambiguity in the def of the geometric entropy, since ξ is not measureable

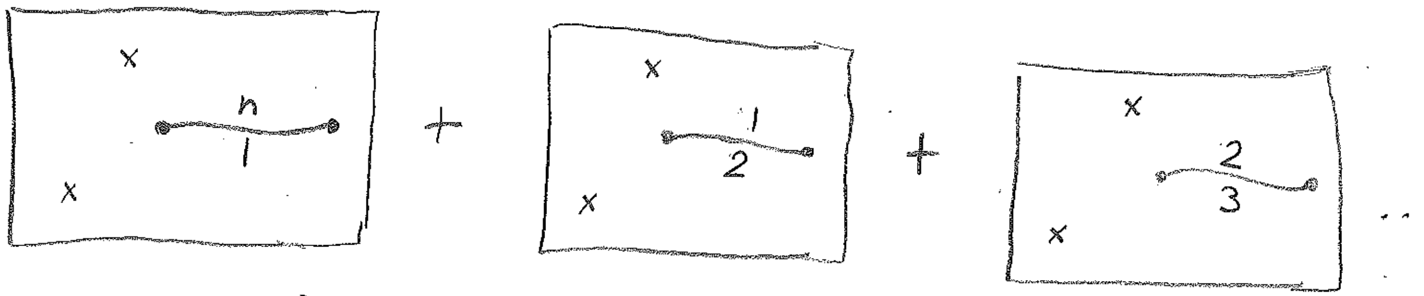
In a gravity thy, makes a physical difference to BH entropy

REPLICA TRICK [*]

What if no $U(1)$ symmetry?

e.g. curved E, unsymmetric sources / operator insertions

Must do replica trick – copy space n times



n identical copies

Defines $Z_n = \mathrm{tr}(\rho^n)$

same formula works: $S = (1 - \cancel{n}\partial_n) \ln Z_n \Big|_{n=1}$

but now we have to analytically continue to noninteger n, $= \lim_{n \to 1} \frac{1}{1-n} \ln \underbrace{\mathrm{tr}(\rho^n)}_{\frac{Z_n}{Z_1^n}}$ Renyi entropy [*] (obeys some but not all properties of S)

(conical method special case where it's easy)

How to continue from integer n?

Seems ambiguous due to e.g. $+ \sin(\pi n m)$

but this blows up exp as $n \to i\infty$

Carlson's Thm. guarantees unique continuation if function does not blow up @ ∞ too fast, as expected for $\mathrm{tr}(\rho^n)$ $\mathrm{Re}(n) \geq 1$

but phase transitions can mess up analyticity... [more details]

Alt. viewpoint: Manifold has $\mathbb{Z}_n$ replica symmetry

add "twist operators" at E which implement twist.

Very useful in 1+1 CFT where they behave just like any local $\mathcal{O}$ for $n \in \mathbb{Z}$

[primary w/ $\Delta = \frac{c}{12}\left(n - \frac{1}{n}\right)$] (1+1 replica conformally flat)

ROSENHAUS - SMOLKIN PERTURBATIVE ?

$$\delta S = -\langle O K\rangle \delta\lambda + \frac{(\delta\lambda)^2}{2}\left(\langle O O K\rangle - \langle O O\rangle\right) + \ldots$$

"first law" where $\mathcal{O}$ is some relevant operator that perturbs Euc. path. fn.

neglects commutators, which are contact terms in Euclidean field thy.

analogous result for marginal operators (e.g. deforming geometry) seems to disagree with Solodukhin's K^2 formula (given earlier)

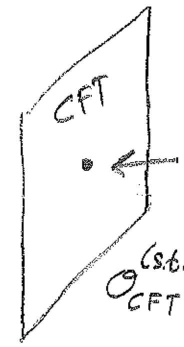
I'M KEEPING THIS BRIEF BECAUSE I PROBABLY WON'T HAVE TIME TO DISCUSS IT.

CONSULT PRIMARY SOURCES FOR MORE DETAIL

HOLOGRAPHIC ENTANGLEMENT ENTROPY

AdS/CFT

[WRITE AdS METRIC?]



Strings/M-theory on $AdS_{d+1} \times F$ $\longleftrightarrow$ CFT_d

$$\mathcal{O}^{(s.b.)}_{CFT} = \lim_{z \to 0} z^{-\Delta} \varphi_{bulk}$$

$\downarrow$ $L_{AdS} \gg \ell_{planck}$ (i.e. $G \to 0$) ($\gg \ell_s$)

$\downarrow$ large "N" (strongly coupled)

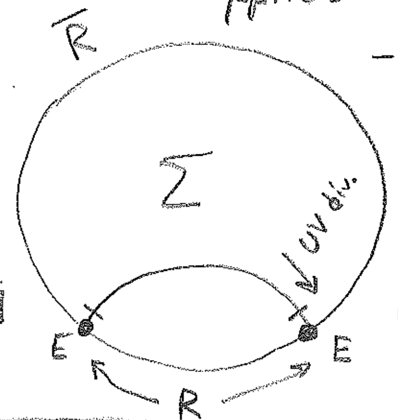
[CHECK THIS]

Supergravity $\longleftrightarrow$ holographic CFT_d

RT formula

[EXAMPLES IN VARIOUS D?]

- formula for leading order piece of entanglement entropy
- applies to static or $t \to -t$ slice Σ
  - people usually ignore F but you don't have to!



[SHOW LOCAL DIV]

new kind of UV regulator (bulk IR cutoff)

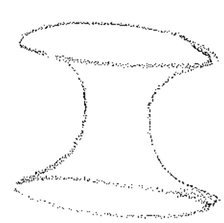
$$\boxed{S_{ent} = \frac{A[\min(R)]}{4G\hbar}}$$

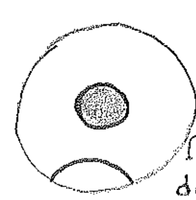
find minimal area surface anchored to E that divides R from $\overline{R}$

$4G\hbar \to \ell_P^{D-2}$ (homology constraint)

move to next

simplest example: take CFT × CFT wormhole





BH example

[mention deconfinement Hawking-Page phase transition]

reduces to BH entropy

examples

d	CFT	bulk	S strong	S weak
2	D1-D5	$AdS_3 \times S_3 \times T_4$	c	c
3	ABJM	$AdS_4 \times S_7$ (M)	$N^{3/2}$ ←	N^2 (UV)
4	$\mathcal{N}=4$ SYM	$AdS_5 \times S_5$ (IIB)	N^2	N^2.
6	(2,0) model	$AdS_7 \times S_4$ (M)	$N^3_{(!)}$ →	N^2 (IR)

c, a protected by SUSY

$\propto 1/G + \mathcal{O}(1)$ corrections

EVIDENCE FROM UNIVERSAL PIECES & CONSISTENCY (E.G SSA)

LM PATH INTEGRAL