1 Problem Set #3

1. Show that the following two expressions for the von Neumann entropy are equivalent:

$$\lim_{n \to 1} \frac{1}{1-n} \ln \mathrm{tr}(\rho^n) = (1 - \beta\partial_\beta) \ln Z|_{\beta=2\pi} \tag{1}$$

2. The "heat kernal regulated" effective action on a smooth spacetime is:

$$S_{\text{eff}} = -\frac{1}{2} \int_{\epsilon^2}^{+\infty} \frac{e^{-sm^2}}{(4\pi s)^{d/2}} \left[\frac{c_0}{s} + c_1 R + \mathcal{O}(s) \right] ds \tag{2}$$

   where m is the mass, ϵ is a regulator with units of distance, and $c_1 = 1/6$ for a minimally coupled scalar field. (The effective action is not local, but the nonlocal aspects are nonperturbative in s and therefore do not appear as terms in this expansion, nor do they contribute to UV divergences.) Use this regulator to calculate the divergent part of the entanglement entropy in a Rindler wedge in 4-dimensional Minkowski spacetime.

3. Consider the left-moving sector $\phi_L(v)$ of a free massless scalar field in 1+1 dimensional Minkowski spacetime, v being a null coordinate. Consider a coherent state formed by acting on the vacuum state $|0>$ with the exponential of the chiral field operator, times a test function f:

$$|\Psi\rangle = \exp\left[\epsilon \int_{t=0} f(x) \nabla_v \phi_L \, dx \right] |0\rangle, \tag{3}$$

   Use the replica trick to calculate the $\mathcal{O}(\epsilon^2)$ piece of the entropy in the interval $x = [0, +\infty]$, at time $t = 0$. (What happens if f is chosen to be imaginary?)

   If you do everything correctly, δS should vanish. This is because the state produced by the exponential turns out to be a coherent state, whose entropy vanishes due to the unitary invariance property of the von Neumann entropy.