1 Problem Set #2

1. Prove that both formulae for the mutual information are equivalent:

$$I_{A,B} = S(A) + S(B) - S(AB) = S(\rho_{AB}|\rho_A \otimes \rho_B). \tag{1}$$

2. Consider the Minkowksi vacuum state of a 2+1 dimensional CFT at one moment of time. Suppose that space at time $t = 0$ is divided into a wedge shaped region $R(\theta)$ lying between two straight lines coming out of the origin at an angle $0 \leq \theta \leq 2\pi$.

   (a) Use dimensional analysis to determine the general form of the entanglement entropy $S(\theta)$ in the region, making sure to correctly parameterize its possible dependence on both a UV and an IR cutoff. Make sure your formula takes into account all conformal symmetries which preserve the region $R(\theta)$, including the spatial inversion given by $\vec{r} \to \vec{r}/|r|^2$.

   (b) Which parts of the expression are universal and which may depend on the choice of regulator scheme?

   (c) What do the properties of the von Neumann entropy (esp. Strong Subadditivity) tell you about how $S(\theta)$ varies as a function of θ?

3. Consider 2+1 dimensional U(1) QED coupled to $2N$ flavors of Dirac fermion, as given by the following weakly coupled action in the UV:

$$I = \int \sum_{i=1}^{2N} \bar{\psi}_i \gamma^\mu (\nabla_\mu - ieA_\mu)\psi_i + \frac{1}{4} F^{\mu\nu} F_{\mu\nu}. \tag{2}$$

   This theory has a $U(2N)$ flavor symmetry. As the theory flows to the IR, it becomes more strongly coupled due to the fact that the charge e is dimensional in 2+1 dimensions, and interesting things may happen. Suppose that there is spontaneous symmetry breaking which produces an effective mass term. If (to preserve time-reversal symmetry) half the fermions gain a mass term with a positive sign, and half gain a mass term with a negative sign, this breaks the flavor symmetry down to a smaller group $U(N) \times U(N)$. Use the F-theorem to estimate the largest number N for which this transition is possible.

   It is a little tricky to deal with the contribution to γ from the photon field, which is logarithmically divergent in the UV (just like the compact scalar field, to which it is dual by p-form duality.) According to arXiv:1310.4886 (Eq. 3.31), the value of γ for the photon is something like:

$$\gamma_{\text{photon, UV}} = 0.852 - \frac{1}{2}\ln(re^2), \tag{3}$$

   and you will need to avoid taking the strict $r \to 0$ limit to get a nontrivial estimated bound.