1 Open Questions (AFAIK)

1. See Challenges #1-#6 in the Headrick review [1].

2. What role, if any, does a nonminimal coupling of a scalar field (a $\xi\phi^2 R$ term in the action) play for entanglement applications in flat space field theory? (E.g. it is known to cancel out in the F-theorem, but does it do so in other applications?)

3. Determine to what extent the "mutual information" across a small gap is a valid regularization procedure for the Entanglement Entropy. (It gives similar results in e.g. free field theory, but in a Topological QFT it seems to be missing the entropy from edge modes.)

4. Understand better the entanglement entropy in lattice Yang-Mills theory, at weak and strong coupling, and at large N. (William Donnelly [2] has identified 3 kinds of contributions to the entropy; two kinds of boundary terms and a bulk term. It would be interesting to determine how these contributions scale with N.) Relate the results to AdS/CFT if possible, being careful to note that there is a phase transition when the theory is sufficiently strongly coupled at the lattice scale, above which the lattice theory no longer flows to the continuum theory in the IR.

5. Figure out from the CFT perspective why there is a weird scaling with N in some holographic field theories. (E.g. $N^{3/2}$ in the ABJM model, N^3 in the $(2,0)$-model.) What are the edge modes in these theories?

6. Calculate the entanglement entropy (or at least the divergent pieces) associated with the graviton or gravitino in an arbitrary region. *Why this is hard:* (a) these field theories are only valid when perturbing around a solution to Einstein (super)gravity (+ cosmological constant), which is not true for the replica manifolds, making the replica trick dubious, (b) since we are varying the metric it might be important to discuss the covariant procedure used to select a particular entangling surface, (c) presumably there are also edge modes, as in the case of Maxwell theory (see Donnelly and Friedel [3]), and (d) actual calculations in the literature, using covariant guage-fixing schemes, have produced discrepant answers. See Appendix A of my Quantum Focussing Paper [4] for a review of the situation.

   The FLM path integral approach [5] should still work, but this only finds the entanglement outside an extremal surface, not a general surface. Is this evidence that the entanglement entropy is ill-defined for a nonextremal entangling surface, for deep quantum gravity reasons? (Notwithstanding the usual expectation from AdS/CFT that the bulk QFT can be defined using Feynman diagrams to any finite order in 1/N).

7. Determine the meaning of the entropy associated with the area (or generalized entropy) of *nonextremal surfaces* in AdS/CFT. (A partial answer is given by the differential entropy formula [6].)

   What is the interpretation of extremal surfaces that don't have minimal area? (Possible partial answer: entwinement [7].)

   Is the area of the causal surface a "coarse-grained version" of the entropy [8]? If so, what is the coarse-graining conjecture in the CFT? (William Kelly and I have a proposal [9], but our evidence was mostly qualitative. In our paper we identified several possible checks of the proposal which have not been done.)

   In the static version of RT, it is natural to suppose that the area of any nonminimal surface corresponds to a "coarse-grained" entropy. But in the covariant version, due to the minus sign in Lorentz signature, there are many surfaces which have *less* area than the extremal surface, which is more difficult to explain in terms of coarse-graining. Can an entropy be assigned to these surfaces?

   Rapahel Sorkin and others have conjectured that the generalized entropy measures the amount of fine-grained information available in a region. Is this conjecture correct? (It is not obviously compatible with AdS/CFT). If it is wrong, what is the correct thing to say?

8. Describe in more detail how it is possible to reconstruct observables in the entanglement wedge from the data in the boundary CFT region. What happens when there is a phase transition and the entanglement wedge suddenly gets a lot bigger?

9. Understand better the quantum corrections to the Ryu-Takayanagi formula. What is the relationship between FLM and the state-counting interpretation of the entropy? Is the Engelhardt-Wall proposal [10] correct at higher orders in 1/N?

10. How can we understand the meaning of the holographic principle in AdS/CFT from the perspective of bulk physics? Is there a more local way of looking the flow of information through the spacetime? Is the RT conjecture explained by some set of degrees of freedom which can be localized on the extremal surface?

    Brian Swingle [11] has proposed that AdS/CFT can be understood on the model of "tensor networks", which are a way of generating states popular in condensed matter theory, that can involve a higher dimensional space. Can these tensor network models be extended to explore (a) sub-AdS scales, and (b) dynamics? Are they just a loose analogy, or can they be turned into a serious quantum gravity model?

11. Does information escape from black holes, and if so, can we (and should we) avoid the firewalls paradox? Is it necessary to give up the linearity of quantum

mechanics to do so?

Can holography be used to reconstruct the interior of black holes? What role does quantum error correction play in this reconstruction? Is there a "phase transition" once the black hole becomes sufficiently mixed with the surroundings?

Is Mark Van Raamsdonk [12] correct that spacetime connectivity is built up out of entanglement?

12. Is the Quantum Focussing Conjecture [4] generally true when gravity is coupled to any reasonable semiclassical field theory? If so, what does it tell us about the statistical mechanics of the quantum gravity microstates? And why would the generalized entropy satisfy an inequality concerning the negativity of its *second* time derivative (with respect to the affine parameter labelling a null coordinate)?

References

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