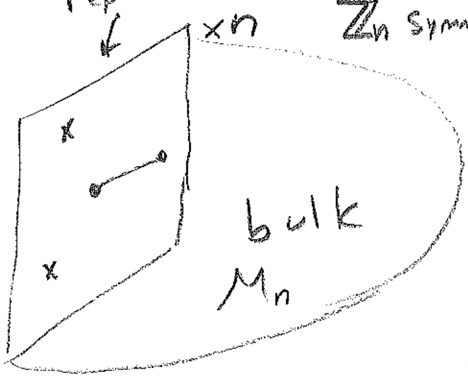
PROOF FROM GRAVITATIONAL PATH INTEGRAL

3.S



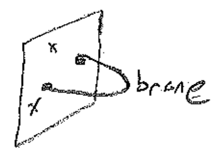
$$Z_{boundary} = Z_{bulk} \qquad \overset{-\ln Z}{I_{eff}}(CFT) = \overset{-\ln Z}{I_{GR}}(bulk)$$

Conical singularity on boundary

might think there should be a singularity in the bulk, but this would violate Einstein.

Instead, find <u>smooth</u> bulk instanton that dominates saddle point w/ specified b.c.

<u>Assume</u> $\mathbb{Z}_n$ symmetry not spontaneously broken



Orbifold $O_n = M_n/\mathbb{Z}_n$ → has original ∂M_1 as boundary
now has conical singularity — "brane" w/ tension
solves $I = I_{GR} + I_{brane}$ $\quad I_{brane} \propto (n-1)A/G$

NB this brane is entirely fictitious — does not really contribute to Z.

Define $I[O_n] = I[M_n]/n$ by stipulation!

[MORE EQ]

now can analytically continue to $n \to 1$ & it's well defined.

[MAKE REDEF OF M_n MORE EXPLICIT]

Note, $n=1$ is original solution to EOM, so $\partial_n (I_{GR} + I_{brane})_{O_n} = 0$

We can switch from $I_{GR} \to -I_{brane} \propto (n-1)\frac{A}{G}[O_n]$

[DONE BETTER ON NEXT PAGE]

use $S_n = \frac{1}{1-n} \ln \frac{Z_n}{Z_1^n}$

[EXTENSION TO COVARIANT CASE]

[Should this be post Wald S?]
[order of limits issue]

36

[MENTION
CASINI-HUERTA-MYERS
DIO STATIONARY
VERSION
OF HYPERBOLIC BH]

$$I[O_n] = \frac{1}{n} I[M_n] \quad @ \text{ integer } n$$

this now <u>defines</u> $I[M_n]$ @ noninteger n

$n=1$ solves EOM for S_{GR}

action of conical deficit

$I_{\text{deficit}} = -I_{\text{brane}}$

if solution to

$I_{\substack{\text{bulk}\\ \text{smooth}}} + I_{\text{def.}} + I_{\text{brane}}$

by varying the area along the brane

$$\text{so } \partial_n\left(I[O_n] - (1-n)\frac{A[O_n]}{G}\right)\Big|_{n=1} = 0$$

$n = 1+\epsilon$

$$\text{so } I(O_{1+\epsilon}) - \epsilon\frac{A[O_{n=1}]}{G} = I(O_1) \left(= I(M_1)\right)$$

[FIXED SIGNS]

$$I(M_{1+\epsilon}) = (1+\epsilon)\, I(O_{1+\epsilon}) = (1+\epsilon) I(M_1) + \epsilon \frac{A[ext]}{G}$$

$$S = +\frac{1}{\epsilon}\Big(I(M_{1+\epsilon}) - (1+\epsilon) I(M_1)\Big) = \frac{A[ext]}{G}$$

- from $\frac{1}{1-n}$
- from $I = -\ln Z$

[NOT KEEPING TRACK OF NUMERICAL FACTORS...]

---

earlier
Fursaev Proof
erroneous as pointed out
by Headrick.

Did conical variation in <u>bulk</u>, but
result not saddle point,
& gets wrong result for Renyi's,
(but right for von Neumann.)

FLM?

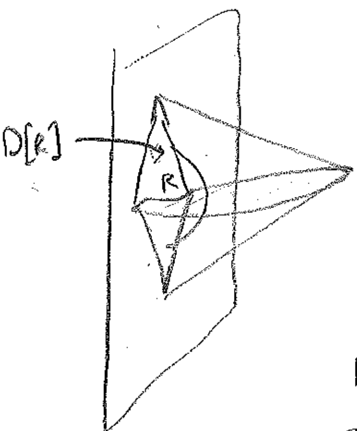
CHECKING SIGNS

$+R \to +\ln Z \to -I$

$I = -\frac{R}{16\pi G}$

BULK RECONSTRUCTION

Suppose we have region R on the boundary

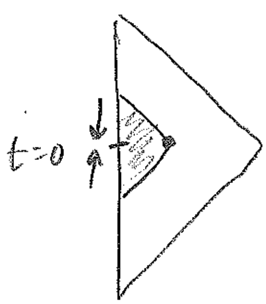


Armed only with ρ_R, how much of the bulk can we reconstruct?

Causal wedge $= I^{-}(D[R]) \cap I^{+}(D[R])$

region for which signals can leave D[R] & return
could be probed thru experiment

HKLL — nonstandard Cauchy problem
- Fourier in transverse directions

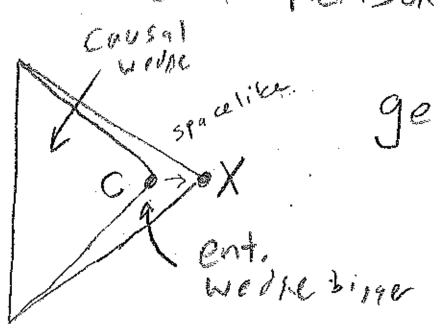


reduce to 1+1 Cauchy evolution

evolve back to $t=0$ using CFT evolution

[OPEN QUESTION IF NO SYM HOW TO GENERALIZE]

BUT GOOD REASONS TO THINK YOU CAN GO FARTHER



generically, X lies deeper in bulk than C

(coincides in special cases like 1 interval AdS)

& $A[C] \geq A[X]$ = whenever analytically known, but A[C] fails SSA, $S[R] = S[\bar{R}]$ etc.

coarse-grained entropy??? — $A[C]$; fine grained S_{CFT} — $A[X]$

A[X] is special, should be at boundary of special region

natural conjecture, can reconstruct all fields in ent wedge.

evidence: consistency conditions

JLMS mod $K_{CFT} = K_{bulk} + \frac{A[ext]}{4G_N}$ + DHW

[QUANTUM ERROR CORRECTION?]

tensor networks: HPPY model & Q error correction (ADH)

if time...

HORIZONS

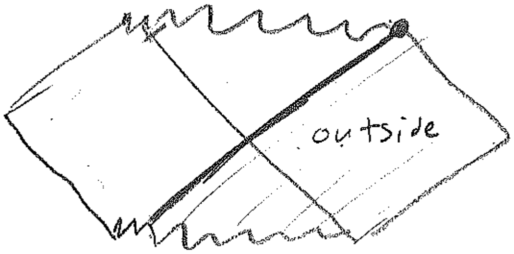
"future
Causal horizon":

boundary of past of locus of points @ fut infinity.

Examples:

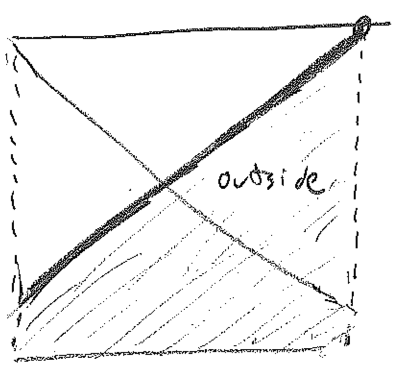
"past CH" = time reverse

BH (+ pert.)



location is still defined
after perturbations

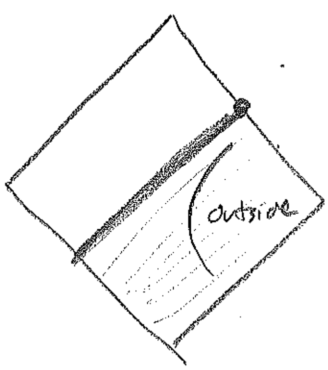
dS (+ pert)



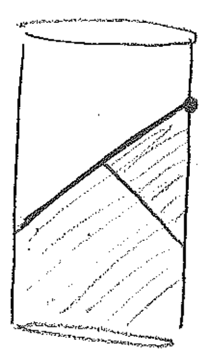
"teleological
boundary
condition"

[DISCUSS APPARENT
HORIZONS?]

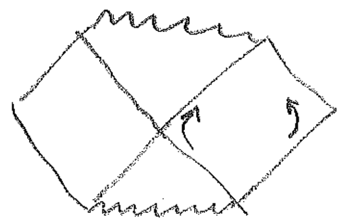
Rindler $R_{1,1} \times R_{D-2}$



Rindler AdS



Black Hole Thermodynamics



Energy $E = M$

Temp $T = \frac{\hbar \kappa}{2\pi}$ where κ = surface gravity = ratio of $\frac{\text{boost @ horizon}}{\text{time @ } \infty}$ for Killing horizon

Entropy $S = \frac{A}{4G\hbar}$ $(c = k_B = 1)$

ZEROTH LAW

T constant @ equilibrium

$\Downarrow$

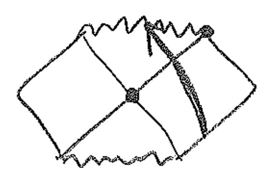
κ constant for stationary Killing horizon

follows from "dominant energy condition" $T_{ab} t^a u^b \geq 0$ (future timelike)

or from sufficient symmetry (static, or axisymmetric & "t-ϕ" reflection symmetric)

ENERGY CONSERVATION

E is encoded in ADM mass @ ∞,
which is conserved due to diffeo symmetry



"FIRST LAW" / CLAUSIUS RELATION

$$dE = T\,dS \left[+ \Omega\, dJ + \phi\, dQ + \ldots\right]$$

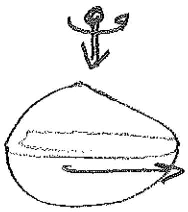
(for linear pert. to Killing horizon from beginning to end)

chemical potentials for angular momentum J, charge Q

Can transform into co-rotating, $\phi = 0$ frame to get $dE = T\,dS$

THIRD LAW

Cannot by any finite process reach "extremal" BH w/ $T=0$ (e.g. from large enough J, Q)



Can't force spin in w/o adding energy

Does NOT imply $S=0$ when $T=0$!

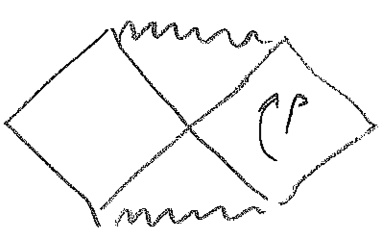
SECOND LAW

Classically, A is increasing if null energy condition

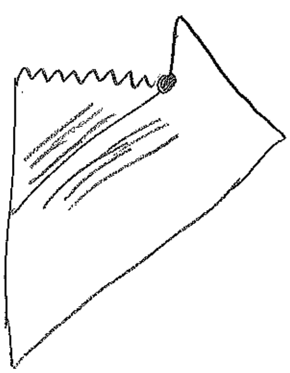
$$T_{ab}k^a k^b \geq 0$$

null vector

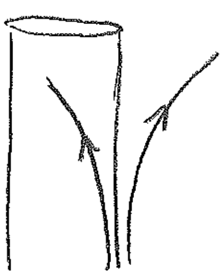
Take QM into account, BH's radiate...



ρ is thermal restricted to exterior in HH state.

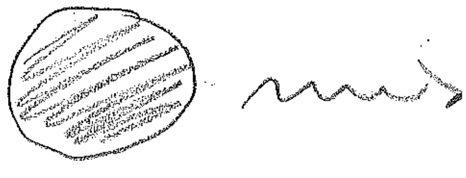


in case of BH w/ forms from collapse, follows from Unruh effect of short distance modes at early times.



So a BH in Unruh state (no incoming rad) will shrink with time, eventually reach Planck size (and probably disappear).

GENERALIZED ENTROPY



BH can radiate to exterior; so what matters is sum of BH ent. and exterior entropy S_{out}

$$S_{gen} = \frac{A}{4G\hbar} + S_{out} + \text{higher-curvature corrections}$$

GSL: $\left.\frac{dS_{gen}}{dt}\right|_{\text{causal horizon}} \geq 0$

Quantum/Thermal

$-\mathrm{tr}(\rho \ln \rho)$, divergent

need regulator ϵ, divergences absorbed into $\frac{1}{G}$, counterterms.

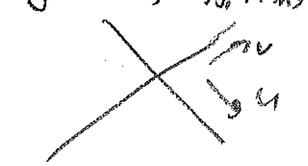
Higher Curv./Stringy

$$I = \int d^D x \sqrt{-g}\, L, \quad L = \frac{R}{16\pi G} + \alpha(R^2 \ldots$$

$\Downarrow$ "Conical" corrections to BH entropy "replica"

Schematic

null coord. $g_{uv} = 1$, i,j transverse



$$S_{BH} = -\frac{2\pi}{\hbar}\int d^{D-2}x \sqrt{{}^{D-2}g}\left[\underbrace{4\frac{\partial L}{\partial R_{uvuv}}}_{\text{Wald, valid stationary, } f(R) \text{ "Noether charge"}} + \underbrace{16\frac{\partial^2 L}{\partial R_{uiuj}\,\partial R_{vkvl}} K^{(u)}_{ij} K^{(v)}_{kl}}_{\text{Solodukhin, FPS, Dong, Camps, Miao...}} + O(K^4) + \ldots\right.$$

$$= -\frac{2\pi}{\hbar}\int_E \sqrt{g}\,\frac{\partial I}{\partial R_{abcd}}\epsilon_{ab}\epsilon_{cd}$$

where E is on horizon

quadratic; $f(\text{Riemann})$ Holographic EE; "splitting problem" derivatives of R

agrees w/ Wall 2nd Law for Killing + 1st order pert. for all higher curv. thy